

Ghosts in the Matter Forms

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Abstract

In this paper we study the matter form of the conformal and super-conformal ghosts action. That is, the ghost fields will be expressed in terms of some scalar and spinor fields. Thus, we obtain a two-dimensional covariant action in the matter form, *i.e.* S_g . The Poincaré-like symmetries and various supersymmetries of this covariant action are analyzed. The signatures 10+2 and 11+3 for the total target space of the superstring theory also will be discussed.

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1 Introduction

In the recent years, string theories can be understood by assuming the existence of higher dimensional target spaces [1, 2, 3, 4, 5, 6]. The eleven-dimensional M-theory [3] and the twelve-dimensional theory known as F-theory [4], are examples of this context. The analysis of the super p -brane scanning allows spacetimes with non-Lorentzian signatures [5]. In other words, there are several models which have more than one “time” coordinate [2, 4, 5, 6]. In particular, twelve-dimensional theories and their invariances with respect to the $SO(10, 2)$ rotations have been investigated [2].

Since the superstring possesses gauge symmetries, namely worldsheet reparametrization invariance, the procedure for the path integral quantization of the superstring is the Faddeev-Popov method. On the other hand, ghosts are quantum fields used to give a functional integral representation of the Faddeev-Popov determinant [7]. They also have an important role in the BRST quantization [8]. If two or more-time world is real, we should be able to formulate the superstring theory in the language of two or more-time physics without the conformal and super-conformal ghosts.

We shall express the action of the super-conformal and conformal ghosts in the covariant form of the matter fields. Therefore, the ghost fields have expressions in terms of the bosonic and fermionic fields. Quantum consistency of the ghosts action in the matter form and initial form will be shown. The matter form of the action enables us to study the symmetries of the theory. Two of these symmetries are $N = 1$ and $N = 2$ supersymmetries. However, in these formulations the superstring lives in the 11+3 or 10+2 dimensional spacetimes without any ghost field.

Besides the $N = 2$ supersymmetry, the theory is invariant with respect to two Poincaré-like symmetries and two other supersymmetries. For each of these symmetries there are two conserved currents. That is, each symmetry is described by the product of two distinct groups.

This paper is organized as follows. In section 2, by introducing some vectors, the super-conformal and conformal ghosts and their action will be expressed in terms of the matter fields. In section 3, the superstring action beyond the dimension ten will be presented. In section 4, Poincaré-like symmetries, bi-supersymmetries and $N = 2$ supersymmetry of the new form of the ghosts action will be studied. In section 5, the signature 11+3 for the total target spacetime of the superstring theory will be discussed.

2 Matter form of the ghosts action

The superstring with the worldsheet supersymmetry has the action

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(G_{\mu\nu} (\eta^{ab} \partial_a X^\mu \partial_b X^\nu - i \bar{\psi}^\mu \rho^a \partial_a \psi^\nu) \right) + S_g, \quad (1)$$

where S_g is sum of the conformal and super-conformal ghosts actions, *i.e.* $S_g = S_{scg} + S_{cg}$. As we know, the spacetime corresponding to the action (1) has the dimension ten with the signature 9+1. Now we proceed to study the covariant matter form of the action S_g .

2.1 The super-conformal ghosts

The super-conformal ghosts have the action

$$S_{scg} = \frac{1}{2\pi\alpha'} \int d^2\sigma (\beta \partial_+ \gamma + \tilde{\beta} \partial_- \tilde{\gamma}), \quad (2)$$

where $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. In this action the fields β and γ (and also $\tilde{\beta}$ and $\tilde{\gamma}$) enter symmetrically, despite the asymmetrical appearance of them. This is due to the flat worldsheet.

Consider the quantities $\{Y^P, \tilde{Y}^P, \partial_+ Z^P, \partial_- \tilde{Z}^P\}$. We express them in terms of the variables $\{\partial_- X^p, \partial_+ X^p, \partial_- \tilde{X}^p, \partial_+ \tilde{X}^p\}$ as in the following

$$\begin{aligned} Y^P &= e^P_p \partial_- X^p, \\ \partial_+ Z^P &= e^P_p \partial_+ X^p, \\ \tilde{Y}^P &= e^P_p \partial_+ \tilde{X}^p, \\ \partial_- \tilde{Z}^P &= e^P_p \partial_- \tilde{X}^p, \end{aligned} \quad (3)$$

where the set $\{e^P_p\}$ denotes the two-dimensional vielbeins with $p, P \in \{1, 2\}$. The other vielbeins (e.g., those that connect Y^P to $\partial_+ X^p$ and $\partial_\pm \tilde{X}^p$) are zero. The equations (3) imply that only four coordinates of the set $\{Y^P, \tilde{Y}^P, Z^P, \tilde{Z}^P\}$ are independent. For example, for the constant vielbeins there are $\partial_+ Y^P = \partial_+ \partial_- Z^P$ and $\partial_- \tilde{Y}^P = \partial_+ \partial_- \tilde{Z}^P$.

We demand the action (2) to be

$$S_{scg} = \frac{1}{2\pi\alpha'} \int d^2\sigma \left(\eta_{PQ} (Y^P \partial_+ Z^Q + \tilde{Y}^P \partial_- \tilde{Z}^Q) \right), \quad (4)$$

where the metric η_{PQ} has the Lorentzian signature, *i.e.* $\eta_{PQ} = \text{diag}(-1, 1)$. This action contains the bosonic fields and has the feature of the action (2). Therefore, the fields of (4) have the roles of the super-conformal ghosts. Expanding the action (4) gives two copies of the action (2). On the other hand, the action (4) has all symmetries of the action (2).

Define the two-dimensional metric G_{pq} as

$$\begin{aligned} G_{pq} &= \sum_{P,Q} (\eta_{PQ} e^P_p e^Q_q) , \\ \eta^{PQ} &= \sum_{p,q} (G^{pq} e^P_p e^Q_q) . \end{aligned} \quad (5)$$

Therefore, the action (4) takes the form

$$S_{scg} = -\frac{1}{8\pi\alpha'} \int d^2\sigma \left(G_{pq} \eta^{ab} (\partial_a X^p \partial_b X^q + \partial_a \tilde{X}^p \partial_b \tilde{X}^q) \right) . \quad (6)$$

This action corresponds to the manifold \mathcal{M}_g with the coordinates $\{X^p, \tilde{X}^p\}$ and the metric

$$\bar{G} = \begin{pmatrix} \frac{1}{2}G_{pq} & 0 \\ 0 & \frac{1}{2}G_{pq} \end{pmatrix} . \quad (7)$$

We call \mathcal{M}_g as the ghosts manifold. The metric G_{pq} provides a background for the string which propagates in this manifold. This form of the action is covariant with respect to the worldsheet indices $\{a, b\}$ and the manifold indices $\{p, q\}$. We shall see that there are some conditions on the fields of the action (6) which give the equality of the degrees of freedom of this action with the action (2).

In a system with the D -dimensional Poincaré symmetry $ISO(D-1, 1)$, the conformal symmetry is $SO(D, 2)$. In fact, when one considers the conformal symmetry, the symmetry $SO(1, 1)$ is added to the original global symmetry. Thus, besides the usual time coordinate, the conformal symmetry introduces another time coordinate. These imply that the ghosts manifold is product of two identical copies of a two-dimensional spacetime. In other words, we have a four-dimensional spacetime with the signature 2+2. We shall see that the 1+1 interpretation for the signature of this spacetime also is possible.

In fact, the action (4) is auxiliary. Since the actions (2) and (4) have the same feature we can write

$$\begin{aligned} \beta \partial_+ \gamma &= \eta_{PQ} Y^P \partial_+ Z^Q , \\ \tilde{\beta} \partial_- \tilde{\gamma} &= \eta_{PQ} \tilde{Y}^P \partial_- \tilde{Z}^Q . \end{aligned} \quad (8)$$

Let V^P and \tilde{V}^P be two unit vectors, *i.e.*,

$$\eta_{PQ} V^P V^Q = \eta_{PQ} \tilde{V}^P \tilde{V}^Q = 1 . \quad (9)$$

Insertion of these vectors in the left-hand sides of the equations (8) leads to the equations

$$\begin{aligned} (\beta V)^T \eta (V \partial_+ \gamma) &= Y^T \eta \partial_+ Z , \\ (\tilde{\beta} \tilde{V})^T \eta (\tilde{V} \partial_- \tilde{\gamma}) &= \tilde{Y}^T \eta \partial_- \tilde{Z} . \end{aligned} \quad (10)$$

One solution of these equations is

$$\begin{aligned}
\beta V &= Y , \\
V \partial_+ \gamma &= \partial_+ Z + U , \\
\tilde{\beta} \tilde{V} &= \tilde{Y} , \\
\tilde{V} \partial_- \tilde{\gamma} &= \partial_- \tilde{Z} + \tilde{U} ,
\end{aligned} \tag{11}$$

where the vectors U^P and \tilde{U}^P are perpendicular to Y^P and \tilde{Y}^P , respectively,

$$\eta_{PQ} Y^P U^Q = \eta_{PQ} \tilde{Y}^P \tilde{U}^Q = 0. \tag{12}$$

However, for the next purposes, we assume they are not perpendicular to V^P and \tilde{V}^P , *i.e.*,

$$\eta_{PQ} V^P U^Q \neq 0 \quad , \quad \eta_{PQ} \tilde{V}^P \tilde{U}^Q \neq 0. \tag{13}$$

Other solutions for the equations (10) are possible. For example, they can be written in the form $(V \partial_+ \gamma)^T \eta(\beta V) = Y^T \eta \partial_+ Z$ and $(\tilde{V} \partial_- \tilde{\gamma})^T \eta(\tilde{\beta} \tilde{V}) = \tilde{Y}^T \eta \partial_- \tilde{Z}$. The solution of these equations is different from (11). We consider only the solution (11).

In terms of the fields of the action (4) and also in terms of the coordinates $\{X^p, \tilde{X}^p\}$ the super-conformal ghosts have the following expressions

$$\begin{aligned}
\beta &= \eta_{PQ} V^P Y^Q = v_p \partial_- X^p , \\
\partial_+ \gamma &= \eta_{PQ} V^P (\partial_+ Z^Q + U^Q) = v_p \partial_+ X^p + v.u , \\
\tilde{\beta} &= \eta_{PQ} \tilde{V}^P \tilde{Y}^Q = \tilde{v}_p \partial_+ \tilde{X}^p , \\
\partial_- \tilde{\gamma} &= \eta_{PQ} \tilde{V}^P (\partial_- \tilde{Z}^Q + \tilde{U}^Q) = \tilde{v}_p \partial_- \tilde{X}^p + \tilde{v}.\tilde{u} ,
\end{aligned} \tag{14}$$

where the vectors v_p and \tilde{v}_p have the definitions

$$\begin{aligned}
v_p &= \eta_{PQ} e^P_p V^Q , \\
\tilde{v}_p &= \eta_{PQ} e^P_p \tilde{V}^Q ,
\end{aligned} \tag{15}$$

similarly for the vectors u_p and \tilde{u}_p . The inner product $v.u$ is defined by $v.u = G_{pq} v^p u^q$. According to the equations (13), $v.u$ and $\tilde{v}.\tilde{u}$ are nonzero. Note that v_p and \tilde{v}_p also are unit vectors, *i.e.* $G^{pq} v_p v_q = G^{pq} \tilde{v}_p \tilde{v}_q = 1$. The equations (14) imply that the super-conformal ghosts can be seen as linear combinations of some scalar fields.

To understand more about the mappings (14), let consider the bosonization of the super-ghosts β and γ ,

$$\beta = e^{-\phi} \partial_- \xi , \quad \gamma = e^{\phi} \eta , \quad \xi = e^{\zeta} , \quad \eta = e^{-\zeta} , \tag{16}$$

where the bosonized super-ghosts are ϕ , ξ , η and ζ [9]. A similar construction for the ghosts b and c was carried out in Ref.[10]. As we see, β depends on the derivative of a field, while γ is independent of any derivative. This also is true for β and γ in the equations (14). For the field γ also see the first equation of (31). Therefore, the equations (14) can be interpreted as a kind of bosonization. Thus, the fields X^1 and X^2 have the following relations with the bosonized super-ghosts ϕ and ζ ,

$$\begin{aligned} v_p \partial_- X^p &= e^{-\phi+\zeta} \partial_- \zeta , \\ v.u + v_p \partial_+ X^p &= \partial_+ e^{\phi-\zeta} . \end{aligned} \quad (17)$$

Similar interpretation also holds for the left-moving fields.

According to the equations (11), it is possible to express the fields $\{X^p, \tilde{X}^p\}$ in terms of the ghost fields

$$\begin{aligned} \partial_- X^p &= v^p \beta , \\ \partial_+ X^p &= v^p \partial_+ \gamma - u^p , \\ \partial_+ \tilde{X}^p &= \tilde{v}^p \tilde{\beta} , \\ \partial_- \tilde{X}^p &= \tilde{v}^p \partial_- \tilde{\gamma} - \tilde{u}^p . \end{aligned} \quad (18)$$

The contravariant vectors are $v^p = e_P^p V^P$, $\tilde{v}^p = e_P^p \tilde{V}^P$, $u^p = e_P^p U^P$ and $\tilde{u}^p = e_P^p \tilde{U}^P$, where the matrix e_P^p is inverse of the vielbein matrix e^P_p . These matrices satisfy the relations $e^P_p e_{P'}^p = \delta^P_{P'}$ and $e^P_p e_P^{p'} = \delta_p^{p'}$.

The solutions of the equations of motion of the action (6) have the general forms $X^p = X_R^p + X_L^p$ and $\tilde{X}^p = \tilde{X}_R^p + \tilde{X}_L^p$. Half of the degrees of freedom of X^p and \tilde{X}^p correspond to the super-conformal ghosts. Now consider the unit vectors V' and \tilde{V}' which are perpendicular to the vectors V and \tilde{V} , respectively. The inner products of the vectors V' and \tilde{V}' with the vectors in the equations (11) lead to the conditions

$$\begin{aligned} \eta_{PQ} V'^P Y^Q &= v'_p \partial_- X^p = 0 , \\ \eta_{PQ} V'^P (\partial_+ Z^Q + U^Q) &= v'_p \partial_+ X^p + v'.u = 0 , \\ \eta_{PQ} \tilde{V}'^P \tilde{Y}^Q &= \tilde{v}'_p \partial_+ \tilde{X}^p = 0 , \\ \eta_{PQ} \tilde{V}'^P (\partial_- \tilde{Z}^Q + \tilde{U}^Q) &= \tilde{v}'_p \partial_- \tilde{X}^p + \tilde{v}'.\tilde{u} = 0 . \end{aligned} \quad (19)$$

These four conditions imply that the number of the degrees of freedom of the actions (2) and (6) are equal. Since the vectors v and \tilde{v} are perpendicular to the vectors v' and \tilde{v}' , respectively, these equations also can be obtained from the equations (18).

2.2 The conformal ghosts

The action of the conformal ghosts is

$$S_{cg} = \frac{1}{2\pi\alpha'} \int d^2\sigma (b\partial_+c + \tilde{b}\partial_-\tilde{c}) . \quad (20)$$

We request this action to be

$$S_{cg} = \frac{i}{2\pi\alpha'} \int d^2\sigma \left(\eta_{PQ} (\Psi_1^P \partial_+ \Theta_1^Q + \Psi_2^P \partial_- \Theta_2^Q) \right) . \quad (21)$$

This action has all symmetries of the action (20). This is due to the fact that, it has two distinct copies of (20).

The Grassmannian variables $\{\Psi_{1,2}^P, \Theta_{1,2}^P\}$ have the following expressions in terms of the worldsheet fermions $\{\psi^p, \theta^p\}$,

$$\begin{aligned} \Psi_1^P &= e^P{}_p \psi_-^p , \\ \Psi_2^P &= e^P{}_p \psi_+^p , \\ \Theta_1^P &= e^P{}_p \theta_-^p , \\ \Theta_2^P &= e^P{}_p \theta_+^p . \end{aligned} \quad (22)$$

Therefore, the auxiliary action (21) can be written in terms of the spinor fields

$$S_{cg} = \frac{i}{4\pi\alpha'} \int d^2\sigma (G_{pq} \bar{\psi}^p \rho^a \partial_a \theta^q) , \quad (23)$$

where $\psi^p = \begin{pmatrix} \psi_-^p \\ \psi_+^p \end{pmatrix}$ and $\theta^p = \begin{pmatrix} \theta_-^p \\ \theta_+^p \end{pmatrix}$ are Majorana spinors. We assumed that the vielbeins are independent of the fields $\{X^p, \tilde{X}^p\}$, and hence they do not depend on the worldsheet coordinates τ and σ . That is, the metric G_{pq} is constant. For matching the degrees of freedom of the actions (20), (21) and (23) see the conditions (30).

Equality of the actions (20) and (21) gives

$$\begin{aligned} b\partial_+c &= i\eta_{PQ} \Psi_1^P \partial_+ \Theta_1^Q , \\ \tilde{b}\partial_-\tilde{c} &= i\eta_{PQ} \Psi_2^P \partial_- \Theta_2^Q . \end{aligned} \quad (24)$$

This is due to the common feature of these actions. Consider the unit vectors W^P and \widetilde{W}^P ,

$$\eta_{PQ} W^P W^Q = \eta_{PQ} \widetilde{W}^P \widetilde{W}^Q = 1 . \quad (25)$$

Furthermore, define the Grassmann valued vectors Λ^P and $\tilde{\Lambda}^P$ with the following properties

$$\eta_{PQ} \Psi_1^P \Lambda^Q = \eta_{PQ} \Psi_2^P \tilde{\Lambda}^Q = 0 , \quad (26)$$

$$\eta_{PQ}W^P\Lambda^Q \neq 0 \quad , \quad \eta_{PQ}\widetilde{W}^P\widetilde{\Lambda}^Q \neq 0. \quad (27)$$

Thus, for example, Λ^P is perpendicular to Ψ_1^P but it is not perpendicular to W^P .

Insert the unit vectors in the left-hand sides of the equations (24). Similar to the relations (10), (11) and (14), we obtain the mappings

$$\begin{aligned} b &= i\eta_{PQ}W^P\Psi_1^Q = iw_p\psi_-^p, \\ \partial_+c &= \eta_{PQ}W^P(\partial_+\Theta_1^Q + \Lambda^Q) = w_p\partial_+\theta_-^p + w.\lambda, \\ \tilde{b} &= i\eta_{PQ}\widetilde{W}^P\Psi_2^Q = i\tilde{w}_p\psi_+^p, \\ \partial_-\tilde{c} &= \eta_{PQ}\widetilde{W}^P(\partial_-\Theta_2^Q + \widetilde{\Lambda}^Q) = \tilde{w}_p\partial_-\theta_+^p + \tilde{w}.\tilde{\lambda}. \end{aligned} \quad (28)$$

The unit vectors w_p and \tilde{w}_p and the Grassmannian vectors λ_p and $\tilde{\lambda}_p$, similar to the equation (15), have definitions in terms of $\{W^P\}$, $\{\widetilde{W}^P\}$, $\{\Lambda^P\}$ and $\{\widetilde{\Lambda}^P\}$, respectively. The equations (27) imply that the inner products $w.\lambda$ and $\tilde{w}.\tilde{\lambda}$ are nonzero. Therefore, according to the equations (28) the conformal ghosts appear as components of some spinor fields. Equivalently, the worldsheet fermions in terms of the conformal ghosts are

$$\begin{aligned} \psi_-^p &= -iw^pb, \\ \partial_+\theta_-^p &= w^p\partial_+c - \lambda^p, \\ \psi_+^p &= -i\tilde{w}^p\tilde{b}, \\ \partial_-\theta_+^p &= \tilde{w}^p\partial_-\tilde{c} - \tilde{\lambda}^p. \end{aligned} \quad (29)$$

Let the unit vectors W' and \widetilde{W}' be perpendicular to the vectors W and \widetilde{W} , respectively. Therefore, we have analog of the equations (19), *i.e.*,

$$\begin{aligned} \eta_{PQ}W'^P\Psi_1^Q &= w'_p\psi_-^p = 0, \\ \eta_{PQ}W'^P(\partial_+\Theta_1^Q + \Lambda^Q) &= w'_p\partial_+\theta_-^p + w'.\lambda = 0, \\ \eta_{PQ}\widetilde{W}'^P\Psi_2^Q &= \tilde{w}'_p\psi_+^p = 0, \\ \eta_{PQ}\widetilde{W}'^P(\partial_-\Theta_2^Q + \widetilde{\Lambda}^Q) &= \tilde{w}'_p\partial_-\theta_+^p + \tilde{w}'.\tilde{\lambda} = 0. \end{aligned} \quad (30)$$

These conditions give the same number of the degrees of freedom for the actions (20), (21) and (23). From the equations (29) also we can obtain these conditions. That is, use the unit vectors w' and \tilde{w}' , which are perpendicular to w and \tilde{w} , respectively.

The physical states, extracted from the action (1), satisfy some conditions. For example, they are BRST-invariant. If we substitute the super-conformal ghosts from (14) and the conformal ghosts from (28), in the BRST charge and physical states, we obtain the equivalent BRST charge and physical states. The equivalent states under the equivalent BRST charge are invariant. This procedure also holds for other conditions on the physical states.

2.3 Quantum consistency

For verifying the quantization, we need the explicit forms of the fields $\gamma, \tilde{\gamma}, c$, and \tilde{c} . The equations (14) and (28) give them as in the following

$$\begin{aligned}\gamma &= v_p X^p + \gamma_0(\sigma^-) + v.u\sigma^+, \\ \tilde{\gamma} &= \tilde{v}_p \tilde{X}^p + \tilde{\gamma}_0(\sigma^+) + \tilde{v}.\tilde{u}\sigma^-, \\ c &= w_p \theta_-^p + c_0(\sigma^-) + w.\lambda\sigma^+, \\ \tilde{c} &= \tilde{w}_p \theta_+^p + \tilde{c}_0(\sigma^+) + \tilde{w}.\tilde{\lambda}\sigma^-, \end{aligned} \tag{31}$$

where the vectors $\{v_p, \tilde{v}_p, w_p, \tilde{w}_p, u_p, \tilde{u}_p, \lambda_p, \tilde{\lambda}_p\}$ are considered independent of the coordinates σ^- and σ^+ .

The functions γ_0 and $\tilde{\gamma}_0$ commute with all fields and c_0 and \tilde{c}_0 anti-commute with all Grassmannian fields. Thus, the canonical quantization of the fields of the bosonic actions (2) and (6) leads to the equations

$$[\gamma(\tau, \sigma), \beta(\tau, \sigma')] = [\tilde{\gamma}(\tau, \sigma), \tilde{\beta}(\tau, \sigma')] = 4\pi i \alpha' \delta(\sigma - \sigma'), \tag{32}$$

$$[X^p(\tau, \sigma), \partial_\tau X^q(\tau, \sigma')] = [\tilde{X}^p(\tau, \sigma), \partial_\tau \tilde{X}^q(\tau, \sigma')] = 4\pi i \alpha' G^{pq} \delta(\sigma - \sigma'). \tag{33}$$

Using the equations (14) and (31) and also the equations of motion of γ and $\tilde{\gamma}$, the quantization (32) leads to the quantization (33) and vice-versa.

In the same way, the canonical quantization of the fields of the actions (20) and (23) are

$$\{c(\tau, \sigma), b(\tau, \sigma')\} = \{\tilde{c}(\tau, \sigma), \tilde{b}(\tau, \sigma')\} = 4\pi i \alpha' \delta(\sigma - \sigma'), \tag{34}$$

$$\{\psi_-^p(\tau, \sigma), \theta_-^q(\tau, \sigma')\} = \{\psi_+^p(\tau, \sigma), \theta_+^q(\tau, \sigma')\} = 4\pi \alpha' G^{pq} \delta(\sigma - \sigma'). \tag{35}$$

According to the equations (28) and (31), these quantizations are the same.

3 Total superstring action

Let express the worldsheet fields X^p and \tilde{X}^p in terms of the new variables x^p and \tilde{x}^p as in the following

$$\begin{aligned}X^p &= A^p{}_q x^q + \tilde{B}^p{}_q \tilde{x}^q, \\ \tilde{X}^p &= B^p{}_q x^q + \tilde{A}^p{}_q \tilde{x}^q. \end{aligned} \tag{36}$$

Apply these relations in the action (6) and then only keep the cross term of the new variables. Therefore, the matrices A , \tilde{A} , B and \tilde{B} should satisfy the conditions

$$\begin{aligned} A^T A + B^T B &= 0 , \\ \tilde{A}^T \tilde{A} + \tilde{B}^T \tilde{B} &= 0 , \\ A^T \tilde{B} + B^T \tilde{A} &= \mathbf{1} . \end{aligned} \tag{37}$$

In other words, the actions (6) and (23) take the form

$$S_g = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(G_{pq}(\eta^{ab} \partial_a x^p \partial_b \tilde{x}^q - i\bar{\psi}^p \rho^a \partial_a \theta^q) \right) . \tag{38}$$

Obtaining the action (6) from the bosonic part of the action (38) gives the relations

$$\begin{aligned} A\tilde{A}^T + \tilde{B}B^T &= 0 , \\ A\tilde{B}^T + \tilde{B}A^T &= \mathbf{1} , \\ \tilde{A}B^T + B\tilde{A}^T &= \mathbf{1} . \end{aligned} \tag{39}$$

These equations are not independent of the conditions (37).

However, the equations of motion, extracted from the action (38), are

$$\partial_a \partial^a x^p = \partial_a \partial^a \tilde{x}^p = \rho^a \partial_a \psi^p = \rho^a \partial_a \theta^p = 0 . \tag{40}$$

To remove the reparametrization invariance of the action (38) we can write it as combination of the matter form and the ghost form

$$\begin{aligned} S_g &= -\frac{\mu}{4\pi\alpha'} \int d^2\sigma \left(G_{pq}(\eta^{ab} \partial_a x^p \partial_b \tilde{x}^q - i\bar{\psi}^p \rho^a \partial_a \theta^q) \right) \\ &\quad + (1 - \mu) \left(S_{scg}[Eq(2)] + S_{cg}[Eq(20)] \right) , \end{aligned} \tag{41}$$

where μ is any real number. Each action in the second line does not have the diffeomorphism invariance.

According to the action (38), the superstring action (1) completely can be written in the matter form

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(G_{\bar{\mu}\bar{\nu}}(\eta^{ab} \partial_a x^{\bar{\mu}} \partial_b \tilde{x}^{\bar{\nu}} - i\bar{\psi}^{\bar{\mu}} \rho^a \partial_a \theta^{\bar{\nu}}) \right) , \tag{42}$$

where the metric $G_{\bar{\mu}\bar{\nu}}$ is defined by

$$G_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} G_{\mu\nu} & 0 \\ 0 & G_{pq} \end{pmatrix} . \tag{43}$$

The coordinates x^μ and \tilde{x}^μ are defined by $x^\mu = \tilde{x}^\mu = X^\mu$. We also defined $\theta^\mu = \psi^\mu$. The metrics $G_{\mu\nu}$ and G_{pq} represent 9+1 and 1+1 signatures, respectively. We shall see that the corresponding spacetime to the action (42) can have the signatures 10+2 and 11+3 . Now we study some symmetries of the action (38).

4 Symmetries of the ghosts action in the matter form

Since the symmetries of the matter part of the total superstring action (1) are known, we concentrate on the symmetries of the ghost part, *i.e.* S_g in the form (38).

4.1 The Poincaré-like symmetries

Consider the global transformations

$$\begin{aligned}\delta x^p &= a^p{}_q x^q + b^p , \\ \delta \tilde{x}^p &= \tilde{a}^p{}_q \tilde{x}^q + \tilde{b}^p , \\ \delta \psi^p &= a^p{}_q \psi^q , \\ \delta \theta^p &= \tilde{a}^p{}_q \theta^q ,\end{aligned}\tag{44}$$

where a_{pq} and \tilde{a}_{pq} are antisymmetric constant matrices. Under these transformations the action (38), for $\tilde{a}_{pq} = a_{pq}$, is symmetric. The resulted current is

$$J_a^{pq} = \frac{1}{4\pi\alpha'} [x^p \partial_a \tilde{x}^q - x^q \partial_a \tilde{x}^p + \tilde{x}^p \partial_a x^q - \tilde{x}^q \partial_a x^p + i(\bar{\psi}^p \rho_a \theta^q - \bar{\psi}^q \rho_a \theta^p)] .\tag{45}$$

The equations of motion (40) imply that this is a conserved current

$$\partial^a J_a^{pq} = 0 .\tag{46}$$

For the translations \tilde{b}^p and b^p the associated currents are

$$\begin{aligned}P_a^p &= \frac{1}{4\pi\alpha'} \partial_a x^p , \\ \tilde{P}_a^p &= \frac{1}{4\pi\alpha'} \partial_a \tilde{x}^p ,\end{aligned}\tag{47}$$

respectively. These currents also are conserved

$$\partial^a P_a^p = \partial^a \tilde{P}_a^p = 0 .\tag{48}$$

For $\tilde{b}^p = b^p$ the action (38) again remains invariant and hence we have the conserved current

$$\mathcal{P}_a^p = \frac{1}{4\pi\alpha'} (\partial_a x^p + \partial_a \tilde{x}^p) .\tag{49}$$

Another symmetry of the action (38) is as follows

$$\begin{aligned}\delta x^p &= a^p{}_q \tilde{x}^q + b^p , \\ \delta \tilde{x}^p &= \tilde{a}^p{}_q x^q + \tilde{b}^p , \\ \delta \psi^p &= a^p{}_q \theta^q , \\ \delta \theta^p &= -\tilde{a}^p{}_q \psi^q .\end{aligned}\tag{50}$$

In these transformations the parameters a_{pq} and \tilde{a}_{pq} can be different. Therefore, there are two types of the group generators. The associated currents are

$$\begin{aligned} j_a^{pq} &= \frac{1}{2\pi\alpha'} (x^p \partial_a x^q - x^q \partial_a x^p + i\bar{\psi}^p \rho_a \psi^q) , \\ \tilde{j}_a^{pq} &= \frac{1}{2\pi\alpha'} (\tilde{x}^p \partial_a \tilde{x}^q - \tilde{x}^q \partial_a \tilde{x}^p + i\bar{\theta}^p \rho_a \theta^q) . \end{aligned} \quad (51)$$

Note that j_a^{pq} corresponds to the parameter \tilde{a}_{pq} , while \tilde{j}_a^{pq} corresponds to a_{pq} . These currents also satisfy the conservation laws

$$\partial^a j_a^{pq} = \partial^a \tilde{j}_a^{pq} = 0 . \quad (52)$$

The current equations for the translation parts are the same as (47)-(49). Since the equations (44) and (50) are similar to the usual Poincaré transformations, we call them as the *Poincaré-like* transformations.

4.2 Worldsheet supersymmetries

The matter part of the action (1) is symmetric under the worldsheet supersymmetry transformations

$$\begin{aligned} \delta X^\mu &= \bar{\epsilon} \psi^\mu , \\ \delta \psi^\mu &= -i\rho^a \partial_a X^\mu \epsilon , \end{aligned} \quad (53)$$

where ϵ is an infinitesimal constant spinor. Now we study the various supersymmetries of the ghosts action (38).

4.2.1 Bi-supersymmetries of the worldsheet

Consider the following transformations

$$\begin{aligned} \delta x^p &= \bar{\eta} \psi^p , \\ \delta \tilde{x}^p &= \bar{\tilde{\eta}} \theta^p , \\ \delta \psi^p &= -i\rho^a \partial_a x^p \eta , \\ \delta \theta^p &= -i\rho^a \partial_a \tilde{x}^p \tilde{\eta} , \end{aligned} \quad (54)$$

where η and $\tilde{\eta}$ are anticommuting constant Majorana spinors. In fact, these are two independent transformations of $(x^p, \psi^p; \eta)$ and $(\tilde{x}^p, \theta^p; \tilde{\eta})$. The associated supercurrents are

$$\begin{aligned} J_a &= \frac{1}{2} G_{pq} \rho^b \rho_a \psi^p \partial_b \tilde{x}^q , \\ \tilde{J}_a &= \frac{1}{2} G_{pq} \rho^b \rho_a \theta^p \partial_b x^q . \end{aligned} \quad (55)$$

They correspond to the parameters η and $\tilde{\eta}$, respectively. Since there are two supercurrents, we call this symmetry as *bi-supersymmetry*. For $\tilde{\eta} = \eta$ the action (38) again remains symmetric. This leads to the current

$$\mathcal{J}_a = \frac{1}{2} G_{pq} \rho^b \rho_a (\psi^p \partial_b \tilde{x}^q + \theta^p \partial_b x^q) . \quad (56)$$

The supercurrents (55) and (56) obey the conservation laws, *i.e.*,

$$\partial^a J_a = \partial^a \tilde{J}_a = \partial^a \mathcal{J}_a = 0 . \quad (57)$$

Besides the transformations (54), the action (38) also is invariant under the following bi-supersymmetry transformations

$$\begin{aligned} \delta x^p &= \bar{\lambda} \theta^p , \\ \delta \tilde{x}^p &= \tilde{\bar{\lambda}} \psi^p , \\ \delta \psi^p &= -i \rho^a \partial_a \tilde{x}^p \tilde{\lambda} , \\ \delta \theta^p &= -i \rho^a \partial_a x^p \lambda , \end{aligned} \quad (58)$$

where λ and $\tilde{\lambda}$ are infinitesimal constant real spinors. These transformations contain two independent parts $(x^p, \theta^p; \lambda)$ and $(\tilde{x}^p, \psi^p; \tilde{\lambda})$. The associated supercurrents, corresponding to the parameters λ and $\tilde{\lambda}$, are

$$\begin{aligned} k_a &= \frac{1}{2} G_{pq} \rho^b \rho_a \theta^p \partial_b \tilde{x}^q , \\ \tilde{k}_a &= \frac{1}{2} G_{pq} \rho^b \rho_a \psi^p \partial_b x^q . \end{aligned} \quad (59)$$

For $\tilde{\lambda} = \lambda$ we obtain analog of the current (56),

$$\mathcal{K}_a = \frac{1}{2} G_{pq} \rho^b \rho_a (\psi^p \partial_b x^q + \theta^p \partial_b \tilde{x}^q) . \quad (60)$$

These supercurrents satisfy the conservation equations

$$\partial^a k_a = \partial^a \tilde{k}_a = \partial^a \mathcal{K}_a = 0 . \quad (61)$$

4.2.2 The $N = 2$ supersymmetry

The $SO(10, 2)$ covariant extension of the superstring is considered. For this we change the worldsheet fermions $\{\psi^p, \theta^p\}$ with $\{\chi^p, \tilde{\chi}^p\}$ as in the following

$$\begin{aligned} \psi^p &= (\tilde{B}^T)^p{}_q \chi^q + (\tilde{A}^T)^p{}_q \tilde{\chi}^q , \\ \theta^p &= (A^T)^p{}_q \chi^q + (B^T)^p{}_q \tilde{\chi}^q . \end{aligned} \quad (62)$$

The matrices in these equations are the same that appeared in the equations (36), (37) and (39). Now we introduce these relations in the action (23). Therefore, according to the action (6), the ghosts action takes the form

$$S_g = -\frac{1}{8\pi\alpha'} \int d^2\sigma \left(G_{pq} [\eta^{ab} (\partial_a X^p \partial_b X^q + \partial_a \tilde{X}^p \partial_b \tilde{X}^q) - i(\bar{\chi}^p \rho^a \partial_a \chi^q + \bar{\tilde{\chi}}^p \rho^a \partial_a \tilde{\chi}^q)] \right). \quad (63)$$

Obtaining the fermionic part, the conditions (37) and (39) have been used.

This action manifestly describes the $N = 2$ supersymmetry. The supersymmetry transformations are

$$\begin{aligned} \delta X^p &= \bar{\epsilon} \chi^p + \bar{\tilde{\epsilon}} \tilde{\chi}^p, \\ \delta \tilde{X}^p &= \bar{\epsilon} \tilde{\chi}^p - \bar{\tilde{\epsilon}} \chi^p, \\ \delta \chi^p &= -i\rho^a \partial_a X^p \epsilon + i\rho^a \partial_a \tilde{X}^p \tilde{\epsilon}, \\ \delta \tilde{\chi}^p &= -i\rho^a \partial_a X^p \tilde{\epsilon} - i\rho^a \partial_a \tilde{X}^p \epsilon. \end{aligned} \quad (64)$$

The worldsheet fermions χ^p and $\tilde{\chi}^p$ form an $SO(2)$ doublet.

In the appearance of $N = 2$ supersymmetry the fields $\{X^p, \chi^p, \tilde{X}^p, \tilde{\chi}^p\}$, in the natural way appeared in the action (63). In other words, all fields in this action have originated from the conformal and super-conformal ghosts. That is, they have not been introduced by hand. Obtaining worldsheet supersymmetry with $N \geq 2$, some extra fields usually are added to action by hand.

In fact, the action (63) shows the other matter form of the ghosts action. In this manner, the manifold \mathcal{M}_g represents two-dimensional spacetime with the coordinates $\{X^p\}$ or $\{\tilde{X}^p\}$ and the signature 1+1. This implies that the total target space for the superstring is 10+2 dimensional spacetime with the coordinates $\{X^\mu, X^p\}$ or $\{X^\mu, \tilde{X}^p\}$. As it was explained, this signature has origin in the conformal symmetry. On the other hand, this space is product of the ten-dimensional spacetime and the two-dimensional manifold \mathcal{M}_g . Note that one direction of development of supersymmetric theories is consideration of 12-dimensional theories with the signature 10+2 [2].

The inverse of the fermions redefinition (62) is

$$\begin{aligned} \chi^p &= A^p_q \psi^q + \tilde{B}^p_q \theta^q, \\ \tilde{\chi}^p &= B^p_q \psi^q + \tilde{A}^p_q \theta^q. \end{aligned} \quad (65)$$

Action of the operators ∂_\pm on the equations (36) and then comparing the resulted equations with (65) give the following pairs of the super-partners

$$X^p \leftrightarrow \chi^p,$$

$$\begin{aligned}
\tilde{X}^p &\leftrightarrow \tilde{\chi}^p , \\
x^p &\leftrightarrow \psi^p , \\
\tilde{x}^p &\leftrightarrow \theta^p .
\end{aligned} \tag{66}$$

These are based on the actions (38) and (63) and their equations of motion. That is, $\partial_+ x^p$ ($\partial_- x^p$) acts in the same way as ψ_+^p (ψ_-^p), and so on.

Note that, application of the transformations (54) and (58) in the equations (36) and (65) does not produce the transformations (64). In other words, the transformations (54), (58) and (64) are independent.

5 The signature 11+3 for the spacetime

Define the block diagonal metric \mathcal{G}_{mn} as

$$\mathcal{G}_{mn} = \begin{pmatrix} G_{\mu\nu} & 0 & 0 \\ 0 & \frac{1}{2}G_{pq} & 0 \\ 0 & 0 & \frac{1}{2}G_{p'q'} \end{pmatrix} . \tag{67}$$

In fact, the metrics G_{pq} and $G_{p'q'}$ are equal. They have been given by the equation (5). Therefore, the action (63) and the matter part of the action (1) can be combined as in the following

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(\mathcal{G}_{mn} (\eta^{ab} \partial_a Z^m \partial_b Z^n - i \bar{\Omega}^m \rho^a \partial_a \Omega^n) \right) , \tag{68}$$

where the generalized coordinates $\{Z^m\}$ and the extended worldsheet spinors $\{\Omega^m\}$ are defined by

$$\begin{aligned}
Z^\mu &= X^\mu , \quad Z^p = X^p , \quad Z^{p'} = \tilde{X}^{p'} , \\
\Omega^\mu &= \psi^\mu , \quad \Omega^p = \chi^p , \quad \Omega^{p'} = \tilde{\chi}^{p'} .
\end{aligned} \tag{69}$$

It is easy to see that this action has the $N = 1$ supersymmetry. The related transformations are analog of (53), *i.e.*,

$$\begin{aligned}
\delta Z^m &= \bar{\varepsilon} \Omega^m , \\
\delta \Omega^m &= -i \rho^a \partial_a Z^m \varepsilon ,
\end{aligned} \tag{70}$$

where the spinor ε is real and constant.

As we explained, the metrics G_{pq} and $G_{p'q'}$ are equal. For each of the metrics G_{pq} and $G_{p'q'}$ the symmetry $SO(1,1)$ is added to the global Poincaré symmetry. In other words,

the conformal symmetry implies that the block diagonal metric \mathcal{G}_{mn} and the coordinates $\{Z^m\}$ describe 14-dimensional spacetime, with the signature 11+3. In fact, the conformal symmetry of the system introduces the extra coordinates.

The line element of the spacetime associated to the action (68) is

$$ds^2 = \mathcal{G}_{mn} dZ^m dZ^n = G_{\mu\nu} dX^\mu dX^\nu + \frac{1}{2} G_{pq} (dX^p dX^q + d\tilde{X}^p d\tilde{X}^q) . \quad (71)$$

On the other hand, this spacetime is product of the two manifolds $M \times \mathcal{M}_g$, where M is the 9+1 dimensional spacetime and \mathcal{M}_g is 2+2 dimensional ghosts manifold.

Note that the $N = 1$ supersymmetry in 11+3 dimensions from the various point of view has been studied. For example, see the Ref.[6].

6 Conclusions

The super-conformal and conformal ghosts action, *i.e.* S_g , was expressed in the covariant matter form. The ghost fields also were expressed in terms of the matter fields. In other words, the conformal ghosts are equivalent to some spinor fields and the super-conformal ghosts also can be represented by some scalar fields. We showed that the quantization of the action S_g in the matter form and in the ghost form are consistent, as expected.

We observed that the bosonic fields of the matter form of the action can be interpreted as additional coordinates of the spacetime. The manifold of these extra dimensions is 1+1 or 2+2 dimensional spacetime. Therefore, the total superstring action corresponds to a twelve or fourteen-dimensional spacetimes with two and three time-directions, respectively.

We studied some symmetries of S_g in the matter form. Under two different transformations, which are similar to the Poincaré transformations, the action is invariant. Each of these symmetries gives the various conserved currents. Furthermore, for this action there are two different bi-supersymmetries. A bi-supersymmetry contains two parameters for the transformations and hence two conserved supercurrents. In addition, we observed that the theory has the $N = 2$ supersymmetry. Finally we obtained the total superstring action, with the $N = 1$ supersymmetry, in the 11+3 dimensional spacetime.

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